

5/24/18 "Live as if you are going to die tomorrow, learn as if you are going to live forever." -Ghandi

HW: Final on Wednesday 6/6

⊗ Wednesday starts
@ 9:19 !!!

$$1) \text{ Length} = \int_a^b \sqrt{1 + (f'(x))^2} \, dx$$

$$f(x) = x^2$$

$$f'(x) = 2x$$

$$\text{Length} = \int_0^2 \sqrt{1 + (2x)^2} \, dx = 4.647$$

$$g(x) = x^3$$

$$g'(x) = 3x^2$$

$$\text{Length} = \int_0^2 \sqrt{1 + (3x^2)^2} \, dx = 8.630$$

$y = x^3$ is longer over $[0, 2]$

2) Antiderivative:

a) $\int 3x^4 + 2 \, dx$

$$f(x) = \frac{3x^5}{5} + 2x + c$$

b) $\int (5\sqrt{x} - 3x) \, dx$

$$\int (5x^{\frac{1}{2}} - 3x) \, dx$$

$$f(x) = \frac{5x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{3x^2}{2} + c$$

$$= \frac{10}{3}x^{\frac{3}{2}} - \frac{3}{2}x^2 + c$$

$$= \frac{10}{3}\sqrt{x^3} - \frac{3}{2}x^2 + c$$

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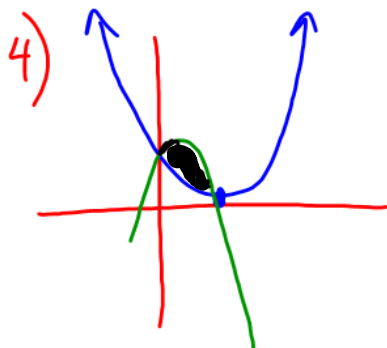
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$$\begin{aligned} 3) \quad a) \quad & \int_0^{\frac{\pi}{2}} \cos(x) \, dx \\ &= \sin x \Big|_0^{\frac{\pi}{2}} \\ &= \sin\left(\frac{\pi}{2}\right) - \sin(0) \\ &= 1 - 0 \\ &= \boxed{1} \end{aligned}$$

$$\begin{aligned} b) \quad & \int_1^2 (3x + x^3) \, dx \\ &= \left[\frac{3x^2}{2} + \frac{x^4}{4} \right]_1^2 \\ &= \left(\frac{3(2)^2}{2} + \frac{2^4}{4} \right) - \left(\frac{3(1)^2}{2} + \frac{1^4}{4} \right) \\ &= (6 + 4) - \left(\frac{3}{2} + \frac{1}{4} \right) \\ &= 10 - \frac{7}{4} \\ &= \boxed{\frac{33}{4}} \end{aligned}$$

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NORMAL FLOAT AUTO REAL DEGREE MP

$$\int_0^2 (-x^2 + 4 - ((x-2)^2)) dx$$

2.666666667

Ans>Frac

 $\frac{8}{3}$

5) $y = x^2 - 4$ $y = 0$ (x-axis)

$$V = \pi \int_a^b f(x)^2 dx$$

NORMAL FLOAT AUTO REAL DEGREE MP

$$\int_{-2}^2 ((x^2 - 4)^2) dx$$

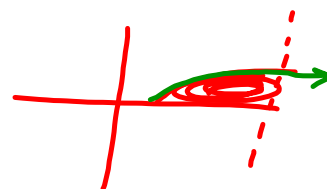
34.13333333

Ans>Frac

 $\frac{512}{15}$

$$V = \frac{512}{15} \pi \text{ units}^3$$

6) $y = \sqrt{x-2}$ $y = 0$ $x = 5$



NORMAL FLOAT AUTO REAL DEGREE MP

$$\int_2^5 ((\sqrt{x-2})^2) dx$$

4.5

$$\text{Volume} = \pi \int_a^b f(x)^2 dx =$$

$$4.5 \pi \text{ units}^3$$

$$7) \text{ slope} = 6x^2 - 2x + 3$$

Point
(1,3)
x y

$$\int 6x^2 - 2x + 3 = \frac{6x^3}{3} - \frac{2x^2}{2} + 3x + C$$

$$2x^3 - x^2 + 3x + C$$

$$3 = 2(1)^3 - 1^2 + 3(1) + C$$

$$3 = 2 - 1 + 3 + C$$

$$f(x) = 2x^3 - x^2 + 3x - 1$$

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$$-1 = C$$

$$8) y^2 - 2x = 3 \quad \text{what is } \frac{dy}{dx} \text{ @ } (3,3)$$

$$\frac{d}{dx}(y^2 - 2x = 3) \quad \begin{array}{l} \text{Implicit} \\ \text{derivative} \end{array} \quad \begin{array}{l} \text{Evaluate} \\ \text{derivative} \end{array}$$

$$2y \frac{dy}{dx} - 2 = 0$$

$$\frac{2y \frac{dy}{dx}}{2y} = \frac{2}{2y} \rightarrow \frac{dy}{dx} = \frac{2}{2y} = \frac{1}{y}$$

Evaluate @ (3,3)

$$\left. \frac{dy}{dx} \right|_{(3,3)} = \left(\frac{1}{3} \right)$$

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$$9) a) \lim_{x \rightarrow 0} \frac{24x^3 - 3x}{12x^3 + x} = \frac{24(0)^3 - 3(0)}{12(0)^3 + (0)} = \frac{0}{0} \text{ Ind.}$$

Try to reduce

$$= \frac{\cancel{3} \cancel{x} (8x^2 - 1)}{\cancel{x} (12x^2 + 1)} = \frac{24x^2 - 3}{12x^2 + 1}$$

Try to plug in again

$$= \frac{24(0)^2 - 3}{12(0)^2 + 1} = \frac{-3}{1}$$

$$\lim_{x \rightarrow 0} \text{ is } (-3)$$

$$b) \lim_{x \rightarrow \infty} \frac{24x^3 - 3x}{12x^3 + x}$$

degrees	limit
$N > D$	∞ or $-\infty$
$D > N$	0
$N = D$	coefficients

Look at
degrees (highest
exponents)

$$\frac{24}{12} = 2$$

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10) $y = \frac{x+2}{2x-1}$

Quotient Rule

$y' = \frac{(2x-1)(1) - (x+2)(2)}{(2x-1)^2}$

need slope

tangent when $x=3$

$$y'_{@x=3} = \frac{(2(3)-1)(1) - (3+2)(2)}{(2(3)-1)^2} = \frac{-5}{25}$$

Point:

$x=3 \quad y = \frac{3+2}{2(3)-1} = \frac{5}{5} = 1$

Slope of tangent = $-\frac{1}{5}$

$(3, 1)$

Equation of tangent line @ $x=3$

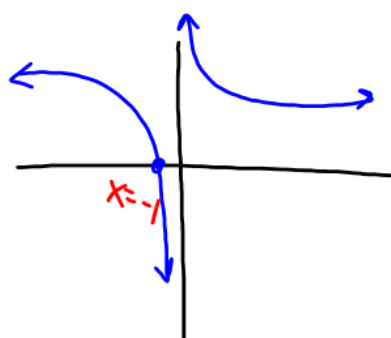
$$y - 1 = -\frac{1}{5}(x - 3)$$

⊗ Normal line is exactly the same with negative reciprocal slope.

11) Concave down, the 2nd derivative is negative.

$$y = 6x^2 + \frac{x}{2} + 3 + \frac{6}{x}$$

$$\frac{6}{x} = 6x^{-1}$$



Concave down
between -1 and 0

$$-1 < x < 0$$

$$y' = 12x + \frac{1}{2} - 6x^{-2}$$

$$y'' = 12 + 12x^{-3}$$

$$y'' = 12 + \frac{12}{x^3}$$

$$0 = 12 + \frac{12}{x^3} \quad \text{DNE @ } x=0$$

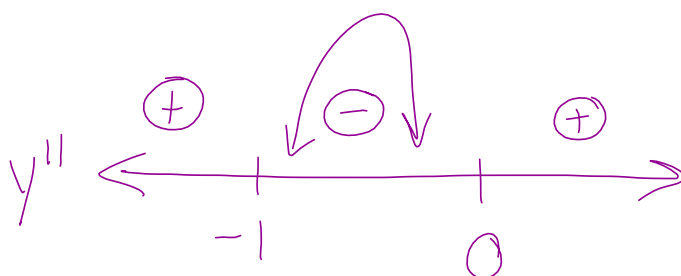
$$-12 = \frac{12}{x^3}$$

$$-12x^3 = 12$$

$$x^3 = -1$$

$$x = -1$$

9

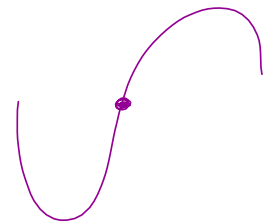


y' tells increasing/Decreasing

$y'=0$ tells max/min

y'' tells concavity up/down

$y''=0$ tells inflection points.



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$$12) \quad f(x) = \frac{x^2-1}{2x} \quad f \quad g$$

$$f'(x) = \frac{2x(2x) - (x^2-1)(2)}{(2x)^2}$$

$$= \frac{4x^2 - 2x^2 + 2}{4x^2} = \frac{2x^2 + 2}{4x^2}$$

Product Rule:
 $f'g + fg'$

Quotient Rule:
 $\frac{gf' - fg'}{(g)^2}$

$$\boxed{\frac{x^2+1}{2x^2}}$$

13) Position $x(t) = \frac{1}{2} \sin(t) + \cos(2t)$

Velocity $x'(t) = \frac{1}{2} \cos(t) - 2 \sin(2t)$

Acceleration $x''(t) = -\frac{1}{2} \sin(t) - 4 \cos(2t)$

$$x''\left(\frac{\pi}{2}\right) = -\frac{1}{2} \sin\left(\frac{\pi}{2}\right) - 4 \cos\left(2 \cdot \frac{\pi}{2}\right)$$

$$= -\frac{1}{2}(1) - 4(-1)$$

$$= \boxed{3.5}$$

$$14) f(x) = e^x \ln x$$

$$f'(x) = e^x \cdot \frac{1}{x} + e^x \ln x$$

$$f'(e) = e^e \cdot \frac{1}{e} + e^e \ln e$$

$$= \frac{e^e}{e} + e^e(1) \approx \boxed{20.729}$$

Recall

$$f(x) = e^x \quad f'(x) = e^x$$

$$f(x) = \ln x \quad f'(x) = \frac{1}{x}$$

$$\ln_e e = 1$$

$$\text{Area} = \int_a^b \text{Top} - \text{bottom} \, dx$$

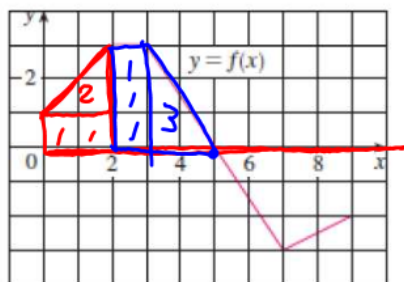
15. The graph of f is shown. Evaluate each integral by interpreting in terms of areas.

- a. $\int_0^2 f(x) \, dx$ Area between $f(x)$ and x -axis between 0 and 2
 b. $\int_2^5 f(x) \, dx$
 c. $\int_0^5 f(x) \, dx$

$$a) = 4$$

$$b) = 6$$

$$c) 4 + 6 = 10$$



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7) Derivative = $6x^2 - 2x + 3$
(Slope)

Looking for
Antiderivative

$$y' = 6x^2 - 2x + 3$$

$$y = \int 6x^2 - 2x + 3 \, dx$$

$$y = \frac{6x^3}{3} - \frac{2x^2}{2} + 3x + c$$

$$y = 2x^3 - x^2 + 3x + c$$

Passes
through

(1, 3)

$$3 = 2(1)^3 - 1^2 + 3(1) + c$$

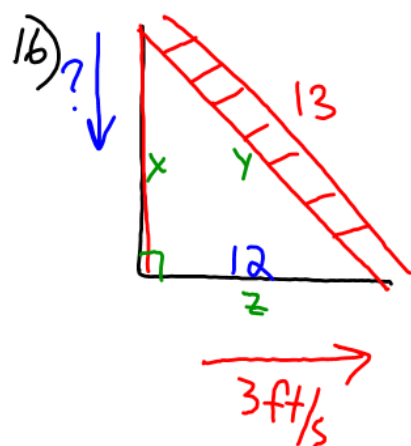
$$3 = 2 - 1 + 3 + c$$

$$3 = 4 + c$$

$$\begin{array}{r} -4 \quad -4 \\ \hline \end{array}$$

$$-1 = c$$

$$y = 2x^3 - x^2 + 3x - 1$$



Know

$$y = 13 \quad \frac{dy}{dt} = 0$$

$$z = 12 \quad \frac{dz}{dt} = 3$$

$$x = 5$$

Want:

$$\frac{dx}{dt}$$

Find x:

$$x^2 + 12^2 = 13^2$$

$$x = 5$$

$$x^2 + z^2 = y^2$$

$$2x \frac{dx}{dt} + 2z \frac{dz}{dt} = 2y \frac{dy}{dt}$$

$$2(5) \frac{dx}{dt} + 2(12)(3) = 2(13)(0)$$

$$10 \frac{dx}{dt} + 72 = 0$$

$$10 \frac{dx}{dt} = -72$$

$$\frac{dx}{dt} = \frac{-72}{10} = -7.2 \text{ ft/sec}$$

Top of ladder
is sliding down
the wall
@ 7.2 ft/sec

\textcircled{R} 18) $f'(x) = \lim_{h \rightarrow 0}$

$$\frac{2(x+h)^2 - 8(x+h) + 5 - (2x^2 - 8x - 5)}{h}$$

$$\frac{\cancel{2x^2} + 4xh + \cancel{2h^2} - \cancel{8x} - 8h + \cancel{5} - \cancel{2x^2} + \cancel{8x} + \cancel{5}}{h}$$

$$\frac{4xh + 2h^2 - 8h}{h} = \frac{\cancel{h}(4x + 2h - 8)}{\cancel{h}} = 4x + 2h - 8$$

$h \rightarrow 0$
 $\boxed{4x - 8}$

$$19) \quad \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \frac{\sqrt{3(x+h)+1} - \sqrt{3x+1}}{h} \cdot \frac{\sqrt{3(x+h)+1} + \sqrt{3x+1}}{\sqrt{3(x+h)+1} + \sqrt{3x+1}}$$

$$\frac{3(x+h)+1 - (3x+1)}{h(\sqrt{3(x+h)+1} + \sqrt{3x+1})} = \frac{\cancel{3x} + \cancel{3h} + \cancel{1} - \cancel{3x} - \cancel{1}}{h(\sqrt{3(x+h)+1} + \sqrt{3x+1})}$$

$$\lim_{h \rightarrow 0} \frac{3}{\sqrt{3(x+h)+1} + \sqrt{3x+1}} = \frac{3}{\sqrt{3x+1} + \sqrt{3x+1}} = \frac{3}{2\sqrt{3x+1}}$$

20. If f and g are the functions shown below. Let $h(x) = f(g(x))$ and $s(x) = f(x)g(x)$.

Find: $h'(1)$ and $s'(1)$

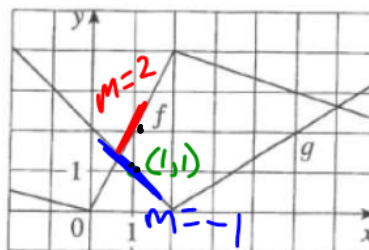
$$g(1) = 1$$

$$h'(x) = f'(g(x)) \cdot g'(x)$$

$$h'(1) = f'(g(1)) \cdot g'(1)$$

$$= f'(1) \cdot g'(1)$$

$$h'(1) = 2 \cdot (-1) = -2$$



$$s'(x) = f(x) \cdot g'(x) + f'(x) \cdot g(x)$$

$$s'(1) = f(1) \cdot g'(1) + f'(1) \cdot g(1)$$

$$= (2) \cdot (-1) + (2)(1)$$

$$= -2 + 2$$

$$= 0$$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	1	2	3
2	5	4	3	4
3	0	6	-1	-2

quotient.

chain

If $n(x) = \frac{f(x)}{g(x)}$, $h(x) = f(g(x))$ find the value of each of the following: a) $n'(2)$ b) $h'(1)$

$$a) n'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

$$n'(2) = \frac{g(2)f'(2) - f(2)g'(2)}{(g(2))^2} = \frac{(3)(4) - (5)(4)}{3^2} = \frac{-8}{9}$$

$$b) h'(x) = f'(g(x)) \cdot g'(x)$$

$$h'(1) = f'(g(1)) \cdot g'(1)$$

$$= f'(2) \cdot (3)$$

$$= (4)(3)$$

$$h'(1) = 12$$

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approaching 1 from left

22. $\lim_{x \rightarrow 1^-} f(x) = -\infty$

23. $\lim_{x \rightarrow 1^+} f(x) = \infty$

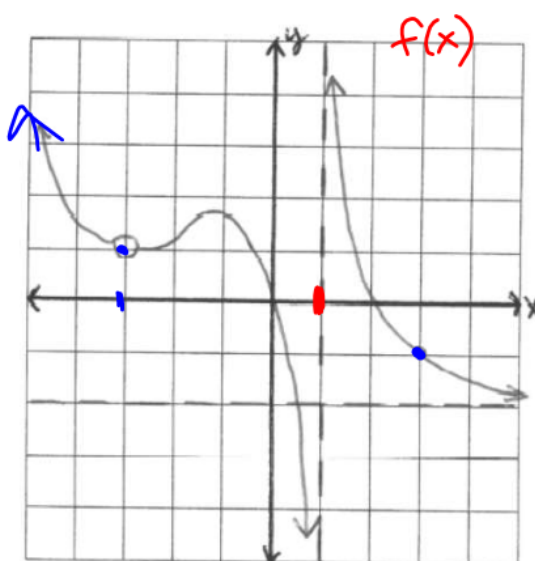
24. $\lim_{x \rightarrow 1} f(x) = \text{DNE}$

25. $\lim_{x \rightarrow -5} f(x) = 1$

26. $\lim_{x \rightarrow 3} f(x) = -1$

27. $\lim_{x \rightarrow -\infty} f(x) = \infty$

28. $\lim_{x \rightarrow \infty} f(x) = -2$



$$29) \quad y = 3x^2 - 2x + 1$$

Tangent @
 $x = -1$

Point:

$$y = 3(-1)^2 - 2(-1) + 1$$

$$y = 6$$

$$(-1, 6)$$

Slope: (Derivative)

$$y' = 6x - 2$$

$$y' = 6(-1) - 2$$

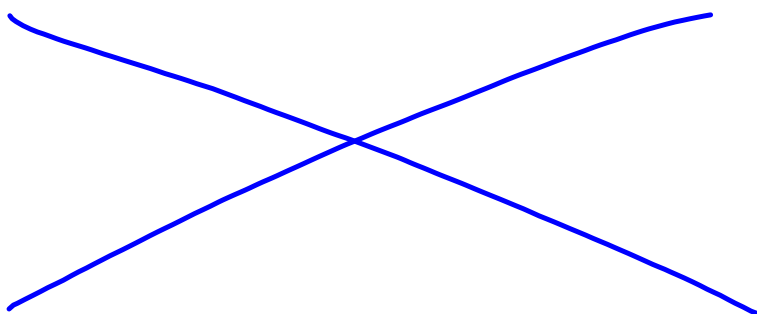
$$y' = -8$$

$$y - 6 = -8(x + 1)$$

29) b) Equation of normal line @ $x = -1$

$$y - 6 = \frac{1}{8}(x + 1)$$

Negative
reciprocal slope
of tangent.



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$$30) s(t) = 2t^3 - 15t^2 + 24t - 10$$

$$v(t) = 6t^2 - 30t + 24$$

$$a(t) = 12t - 30$$

$$a) s(3) = 2(3)^3 - 15(3)^2 + 24(3) - 10 = -19$$

$$b) v(3) = 6(3)^2 - 30(3) + 24 = -12$$

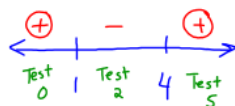
$$c) a(3) = 12(3) - 30 = 6$$

d) Slowing down
b/c $v(3)$ and $a(3)$
have different signs.

e) At rest when $v(t) = 0$

$$\begin{aligned} 0 &= 6t^2 - 30t + 24 \\ &= 6(t^2 - 5t + 4) \\ &= 6(t-4)(t-1) \end{aligned} \quad \begin{matrix} t=4 \\ t=1 \end{matrix}$$

f) Moving \rightarrow when $v(t)$ is positive



Moving Right $[0, 1) \cup (4, \infty)$

g) Used to be tough b/c we had to calculate distance over intervals of left and right motion and then add them.

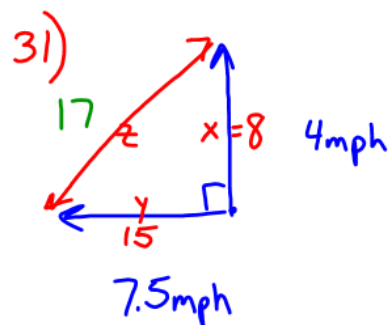
$$\text{Distance} = \int_a^b |v(t)| dt \quad \begin{matrix} \text{absolute} \\ \text{value to} \\ \text{get rid of} \\ + \text{ and } - \text{ velocity} \end{matrix}$$

$$\text{Distance} = \int_0^3 |6t^2 - 30t + 24| dt$$

$$\text{Dist} = 31$$

$$|s(0) - s(1)|$$

$$+ |s(1) - s(3)|$$



$$\begin{aligned} 8^2 + 15^2 &= z^2 \\ 64 + 225 &= z^2 \\ 289 &= z^2 \\ 17 &= z \end{aligned}$$

Have:

$$\begin{aligned} x &= 8 & \frac{dx}{dt} &= 4 \\ y &= 15 & \frac{dy}{dt} &= 7.5 \\ z &= 17 \end{aligned}$$

8.5 mph

$$x^2 + y^2 = z^2$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$$

$$2(8)(4) + 2(15)(7.5) = 2(17) \frac{dz}{dt}$$

$$64 + 225 = 34 \frac{dz}{dt}$$

$$\frac{289}{34} = 34 \frac{dz}{dt}$$

8.5 = $\frac{dz}{dt}$

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$$32) \quad \text{slope} \quad x^2 - xy = y^2 + 1$$

$$\frac{d}{dx}(x^2 - xy = y^2 + 1)$$

$$2x - (x \frac{dy}{dx} + 1y) = 2y \frac{dy}{dx}$$

$$2x - x \frac{dy}{dx} - y = 2y \frac{dy}{dx}$$

$$2(2) - 2 \frac{dy}{dx} - 1 = 2(1) \frac{dy}{dx}$$

$$3 - 2 \frac{dy}{dx} = 2 \frac{dy}{dx}$$

$$+ 2 \frac{dy}{dx} \quad + 2 \frac{dy}{dx}$$

$$\frac{3}{4} = \frac{4 \frac{dy}{dx}}{4}$$

$$\frac{dy}{dx} = \frac{3}{4}$$

$$y - 1 = \frac{3}{4}(x - 2)$$

$$y = 1 \quad \underline{QI}$$

point:

$$x^2 - x(1) = 1^2 + 1$$

$$x^2 - x = 2$$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

$$x = 2 \quad x = -1$$

QI reject not QI

$$(2, 1)$$

Point

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Test 2

#8

$$y = \cos(2x)$$

$$\left[0, \frac{\pi}{2}\right]$$

$$y' = -\sin(2x) \cdot 2$$

$$\text{Length} = \int_0^{\pi/2} \sqrt{1 + (-2\sin(2x))^2} \, dx$$

$$= \boxed{2.635 \text{ units}}$$